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Properties of Canonical-Laplace Transform

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Abstract: The Integral transform is a useful tool for optical analysis and signal processing. In this paper we have defined canonical-Laplace transform and have also proved properties of kernel and linearity property.

Keywords: Integral transform, canonical transform, canonical cosine and sine transforms, Fourier transform, fractional Fourier transform, Laplace transform, testing function space.

1. **INTRODUCTION:** The Fourier analysis is undoubtedly the one of the most valuable and powerful tools in signal processing, image processing and many other branches of engineering sciences [4],[5],[10]the fractional Fourier transform, a special case of linear canonical transform is studied through different analysis .Almeida[1],[2].had introduced it and proved many of its properties . The fractional Fourier transform is a generalization of classical Fourier transform, which is introduce from the mathematical aspect by Namias at first and has many applications in optics quickly[9]. The definition of Laplace transform with parameter p of $f(x)$ denoted by

$$L[f(x)]=F(p)$$

2.
$$L[f(x)]=\int_0^\infty e^{-px}f(x)$$

And definition of canonical transform with parameter s of $f(t)$ denoted by

$$\{CTf(t)\}(s)=\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}e^{-i\left(\frac{s}{b}\right)t}e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)$$

The definition of canonical-Laplace transform is given in section 2. Some properties related to kernel in section 3. Section 4 is proved linearity property. The notation and terminology as per Zemanian [11],[12]. Gelfand-Shilov [3].S.B.Chavhan [6],[7],[8].

2. DEFINITION CANONICAL-LAPLACE TRANSFORMS:

The definition of Laplace transform with parameter p of $f(x)$ denoted by $L[f(x)]=F(p)$

$$L[f(x)]=\int_0^\infty e^{-px}f(x)$$

The definition of Laplace transform with parameter s of $f(t)$ denoted by

$$\{CTf(t)\}(s)=\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}e^{-i\left(\frac{s}{b}\right)t}e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}f(t)$$

The definition of conversional canonical -Laplace transform is defined as

$$CLT\{f(t,x)\}(s,p)=\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{-\infty}^{\infty}\int_0^{\infty}e^{-i\left(\frac{s}{b}\right)t}e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{-px}f(t,x)\,dxdt$$

3. PROPERTIES OF KERNEL:.

ernel of canonical-Laplace transform satisfied following properties.

1) $K_C(t,s)\,K_L(x,p)=K_C(s,t)\,K_L(p,x)$ if $a=d$

2) $K_C(t,s)\,K_L(x,p)\neq K_C(s,t)\,K_L(p,x)$ if $a\neq d$

3) $K_C(-t,s)\,K_L(x,p)=K_C(t,-s)\,K_L(x,p)$

(4) $K_C(-t,s)\,K_L(-x,p)\neq K_C(t,s)\,K_L(x,p)$

(5) $K_C(-t,s)\,K_{C_2}(-x,w)=K_C(t,-s)\,K_L(x,-p)$

(6) $K_C(-t,-s)\,K_L(-x,-p)=K_C(t,s)\,K_L(x,p)$

Proof: (1) By using definition of Canonical-Laplace Transform

$$K_C(t,s)\,K_L(x,p)=K_C(s,t)\,K_L(p,x) \quad \text{if} \quad a=d$$

$$K_C(t,s)K_L(x,p)=\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{-i\left(\frac{s}{b}\right)t}e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{-px}\text{..........}(i)$$

$$K_C(s,t)K_L(p,x)=\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{-i\left(\frac{s}{b}\right)t}e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{-px}\text{..........}(ii)$$

From equation (i) and (ii) result is proved if a=d

Other properties of kernel are same proof.

4. LINEARITY PROPERTY :

If C_1, C_2 are constants and f_1, f_2 are functions of $t\,\&\,x$ then

$$\{CLT\left(C_1f_1(t,x)+C_2f_2(t,x)\right)\}(s,p)=C_1\{CLTf_1(t,x)\}(s,p)+C_2\{CLT\,f_2(t,x)\}(s,p)$$

Proof: We known that $\{CLTf(t,x)\}(s,p)=\langle f(t,x),K_C(t,s)\,K_L(x,p)\rangle$

$$\therefore\{CLT\left[C_1f_2(t,x)+C_2f_2(t,x)\right]\}(s,p)=\langle C_1f_1(t,x)+C_2f_2(t,x),\ K_C(t,s)K_L(x,p)\rangle$$

$$=\langle C_1f_1(t,x),\ K_C(t,s)K_L(x,p)\rangle+\langle C_2f_2(t,x),\ K_C(t,s)K_L(x,p)\rangle$$
$$=C_1\langle f_1(t,x),\ K_C(t,s)K_L(x,p)\rangle+C_2\langle f_2(t,x),\ K_C(t,s)K_L(x,p)\rangle$$
$$=C_1\{CLT\,f_1(t,x)\}(s,p)+C_2\{CLT\,f_2(t,x)\}(s,p)$$
$$\therefore\{CLT\left(C_1f_1(t,x)+C_2f_2(t,x)\right)\}(s,p)$$
$$=C_1\{CLT\,f_1(t,x)\}(s,p)+C_2\{CLT\,f_2(t,x)\}(s,p)$$

. CONCLUSION:

In this paper canonical-Laplace is generalized in the form the distributional sense, and proves some sults related to kernel and linearity properties are discussed.

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